PCM Transmission in the Exchange Plant

By M. R. AARON

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Attention is focused on the choice of a code for transmitting a PCM signal in the exchange plant via paper-insulated cable pairs. Inter- and intra-system crosstalk via near-end coupling is the principal source of interference. A code and repeater structure for reconstructing this code to minimize the effect of crosstalk into the timing channel of a reconstructive repeater is emphasized. It is shown that the conventional self-timed unipolar repeater is not suited to this environment. Several pseudo-ternary codes are developed and surveyed for overcoming crosstalk interference. A bipolar code and a repeater for reconstructing this code are chosen for a variety of reasons. The repeater features nonlinear timing wave extraction and complete retiming and pulse width control, and its realization is discussed in detail in a companion paper by J. S. Mayo.[3]

I. INTRODUCTION

The choice of a modulation method or code for processing a signal preparatory to transmission over a communication channel depends on a multiplicity of conflicting parameters. All of the technical factors must be considered to arrive at a system that serves a need at a justifiable cost. Generally a few of the characteristics of the transmission medium stand out to limit performance. In the situation at hand — pulse transmission at high rates in the exchange plant — near-end crosstalk between cable pairs is the principal transmission deterrent. Examination of the factors that enter into the choice of a simple code and a repeater to reconstruct this code, to combat crosstalk interference, is the principal objective of this paper. Stated another way, we are seeking a transmission scheme that maximizes the number of pairs in a cable that can be used without pair selection.

1.1 *Road Map*

Before we attack this objective it is profitable for later comparisons to provide some system background, to review the functions involved in a PCM reconstructive repeater, and to define terminology. Though the bulk of this introductory material has been covered in the literature, we introduce some new approaches and concepts. It is essential to a clear definition of the problem and our chosen solution. Most of this material is covered in Sections 1.2 through 2.4. Some preliminary comparisons are made in Section 2.5 that contribute to the choice of complete retiming and regeneration. The crosstalk problem is brought into focus in Section III. In Section IV it is shown tbat the conventional pulse-absence of pulse method for transmitting binary PCM is not suited to the present application. Section V contains a survey and evaluation of various pseudo-ternary codes that minimize the problem of crosstalk into the timing channel of a repeater. Reasons for choosing a self-timing bipolar repeater are enumerated at the end of this section. Section VI is devoted to a few words on equalization and equalization optimization. The final section deals with some implications of the interaction of the bipolar repeater with the actual exchange plaut.

1.2 *System Background*

1.2.1 *General*

Other papers [1,2] in this issue have shown that the signal to be transmitted over 22-gauge paper-insulated cable pairs is a 1.544-mc pulse train. Repeaters to reconstruct this pulse train are spaced throughout the medium with spacing dictated by economics, near-end crosstalk coupling and impulse noise. Since loading coils must be removed from voice-frequency pairs prior to pulse transmission, it is economically and administratively desirable simply to replace the loading coils with repeaters. The most common loading coil spacing is nominally 6000 feet, thereby dictating a repeater spacing equal to an integer multiple of 6000 feet. Twelve thousand foot spacing is precluded by near-end crosstalk considerations, as seen later. Therefore the nominal spacing between line repeaters is 6000 feet. Effective variation of electrical length about the nominal is constrained to be ± 500 feet by the use of line-build-out networks. The importance of this assumed constraint on repeater spacing cannot be overemphasized. In effect it removes repeater spacing as a system parameter to be used to combat crosstalk as far as this study is concerned. For example, with shorter repeater spacing the crosstalk problem could

be substantially reduced. The economic penalty could be countered by raising the channel capacity above 24 channels. This of course, would increase the bit rate and bring the crosstalk problem back into prominence. The choice of a transmission scheme most tolerant to crosstalk under this new situation might differ from that chosen for the 6000-ft repeater spacing.

In the vicinity of a central office another source of interference becomes important, namely impulse noise due to switching transients coupled from pair to pair via crosstalk coupling. To combat impulse noise, repeaters adjacent to an office are spaced nominally 3000 feet from the office.

It should be emphasized that with the above spacings and practical signal levels, thermal noise is not a problem.

1.2.2 *Reconstructive Repeaters*

It is well known that the salient feature of PCM transmission is the ability to reconstruct the transmitted pulse train after it has traveled through a dispersive, noisy medium. We will call such repeaters reconstructive repeaters, as opposed to the more commonly used name of regenerative repeaters, since regeneration in the circuit sense is only one facet in reconstructing pulse trains. There are basically three functions that must be performed by such a repeater, namely the three R's—*reshaping*, *retiming*, and *regeneration*.* This operational breakdown is depicted schematically in Fig. 1(a) where the pulse train is traced from repeater to repeater. For purposes of illustration, we assume that the transmitted pulse train at (b) consists of a series of pulses and spaces representing the binary 1 and 0 respectively. This is called a unipolar pulse train, and eventually we will show that this type of code is not suited to the exchange plant environment. However, at this point it suffices for our simple explanations. After transmission over 6000 feet of cable, the high-frequency content of each pulse is severely attenuated and the received pulse train is corrupted by additive interference. The spread-out, "noisy" train appears at (c) (cable output).

The primary function of the first repeater block, *reshaping*, is to shape the signal and raise its level to the point where a pulse vs no pulse decision can be made.

This process involves the inevitable compromise between interference

* *Pulse regeneration* as defined by the American Standards Association in ASA C-42, Group 65 (definition 65.02-102) includes all of the three R's. However, for our purposes the three-way split is more convenient.

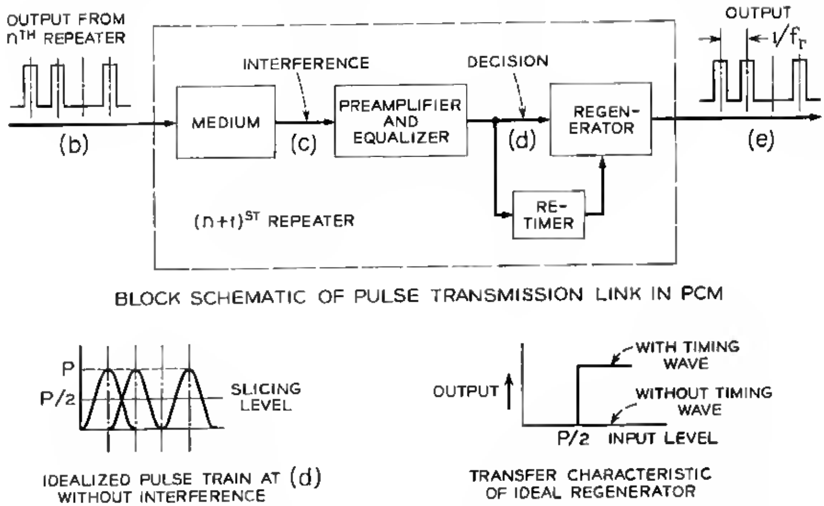

Fig. 1 — PCM transmission: (a) block schematic of pulse transmission link in PCM, (b) idealized pulse train at (d) without interference, (c) transfer characteristic of ideal regenerator.

reduction and pulse resolution.* We will examine reshaping in minor detail in this paper. A more extensive coverage is given in the paper by Mayo,[3] which goes into the details of the realization of the bipolar repeater.

Final reconstruction of the pulse train at d (point of decision) is accomplished by the simultaneous operation of *regeneration* and *retiming.* For purposes of this preliminary discussion we will assume that the regenerator takes the form of a simple threshold detector. The regenerator is enabled when the incoming signal plus interference exceeds the threshold *and* when the timing wave at the output of the retimer has the proper amplitude, or polarity. Other types of regenerators in which regeneration is only partially accomplished in a single repeater are known as partial regenerators.[4] Several partial regenerators in tandem approach complete regeneration. Partial regeneration will not be considered further since complete regeneration can be approached closely with available circuitry (i.e., blocking oscillators) at the frequencies of interest. This situation does not prevail at microwave frequencies.[4]

The purpose of the retimer is threefold: (1) to provide a signal to

* Readers schooled in the lore of digital transmission will recognize the reshaping network as that circuit which, in conjunction with the line and transmitted pulse shape, determines the "eye" picture, i.e., a snapshot of all possible pulse combinations that contribute to each time slot. See Ref. 3.

sample the pulse train where its peak is expected, i.e., where the signal-to-interference ratio is best, (2) to maintain the proper pulse spacing and reduce pulse jitter to minimize distortion in the receiving terminal, and (3) to allow the timing signal to be used to turn off the regenerator to maintain proper pulse width.

In the above manner, provided the signal-to-interference ratio is sufficiently high and the time jitter sufficiently low, the reconstructed pulse train at e is *almost* a replica of the original.

II. TIMING

Since timing plays an important role in PCM transmission, we will introduce and classify methods for *launching*, *extracting* and *using* the timing wave. In addition, we will summarize the sources of timing jitter. Finally, we will make a preliminary comparison of various timing techniques with respect to sources of jitter exclusive of crosstalk. After the initial comparison is made we will characterize the medium and concentrate on the crosstalk problem.

2.1 *Timing Generation*

Methods for launching and extracting timing information can be classified broadly according to whether or not an extra pair is used for transmitting the timing signal. These broad divisions are further subdivided in Table I.

The self-timing category in Table I has been explored in several papers. For this class, the timing wave is obtained by processing the information-bearing pulse train by either linear or nonlinear means or both. Linear extraction, as exemplified by the work of Wrathall,[5] Sunde,[6] and Bennett,[7] is used when the power spectral density of the transmitted pulse train has a discrete line spectrum with a component at the pulse repetition frequency. We will display other pulse trains in which a discrete line at submultiples of the bit rate is produced that can be used

Table I — Timing Classification

1. Same-pair timing
   a. Timing from information-bearing pulses, generally called self timing.
      (1) Linear extraction — forward or backward
      (2) Nonlinear extraction — forward or backward
   b. Timing wave added
      (1) Without spectral null
      (2) With spectral null
   c. Hybrid or dual-mode timing
2. Separate-pair timing

for timing. In other words, the pulse train may have a periodic average value with a fundamental component at frequencies below the bit frequency. This timing component is obtained by exciting any one of several circuits or combinations thereof with the transmitted pulse train. Circuit approaches will be mentioned later.

Self timing can also be employed when the discrete line at the bit rate is absent. Bennett[7] discussed one method for achieving this, and we will consider other possibilities in a later section. Obviously, nonlinear techniques must be used here that take advantage of the underlying phase structure of the pulse train.

When the timing wave is obtained from the transmitted pulse train, it is known as forward-acting. Backward-acting timing indicates that the timing wave is obtained from the reconstructed pulse train at the repeater output and fed back to an internal point in the repeater. We will cover the relative merits of these two approaches when we discuss methods for using the timing wave.

Category (b) under same-pair timing in Table I has not been covered previously in the literature. In this approach a separate sinusoidal signal is added to the transmitted or reconstructed pulse train and extracted at the next repeater. The added timing wave may be used to augment the timing component inherent in the pulse train to insure adequate timing when the pulse train is sparse. Alternatively, the added timing wave can assume the entire timing burden. This may be achieved by any of several methods. First, the energy in the pulse train in the neighborhood of the bit rate can be eliminated by an appropriate filter prior to the addition of the timing wave. Another approach is to use a pulse transmission scheme in which the power spectral density has a null at the bit frequency. A timing wave can be inserted in the resulting slot. Spectral nulls at the bit rate may be obtained by pulse shaping or by converting the binary pulse train to a three-level code, as we shall demonstrate. With the latter method it is possible to produce spectral nulls at submultiples of the bit frequency.

The last same-pair timing category of Table I is really a transition class that leads into separate timing. Dual-mode timing involves the use of self timing in one direction of transmission and the simultaneous use of this timing wave for timing the other direction of transmission. The slave system can use a transmission scheme that has a spectral null at the timing frequency. This approach eliminates the near-end crosstalk (NEXT) interference at the timing frequency.

The final category in Table I is self-explanatory. Further comparison of all these schemes is deferred until we can bring some other factors to bear.

2.2 *Methods of Using Timing Wave*

Just as in the case of regeneration, retiming can be classified as complete or partial. In this section these and related terms, as well as some of those used previously, are given more concrete significance with the aid of the diagrams in Fig. 2.

The simplest scheme to instrument is the partial retiming approach used by Wrathall and analyzed by Sunde. In this approach, the peak of the recovered timing wave (clock) is pinned to ground or some other convenient reference and added to the incoming pulse train, as shown in Fig. 2(a). When the signal-plus-timing wave exceeds the threshold level the regenerator is fired. Obviously, as the timing wave amplitude becomes larger, sampling of the input pulse occurs closer and closer to the pulse peak where the signal-to-interference ratio is best. The timing wave

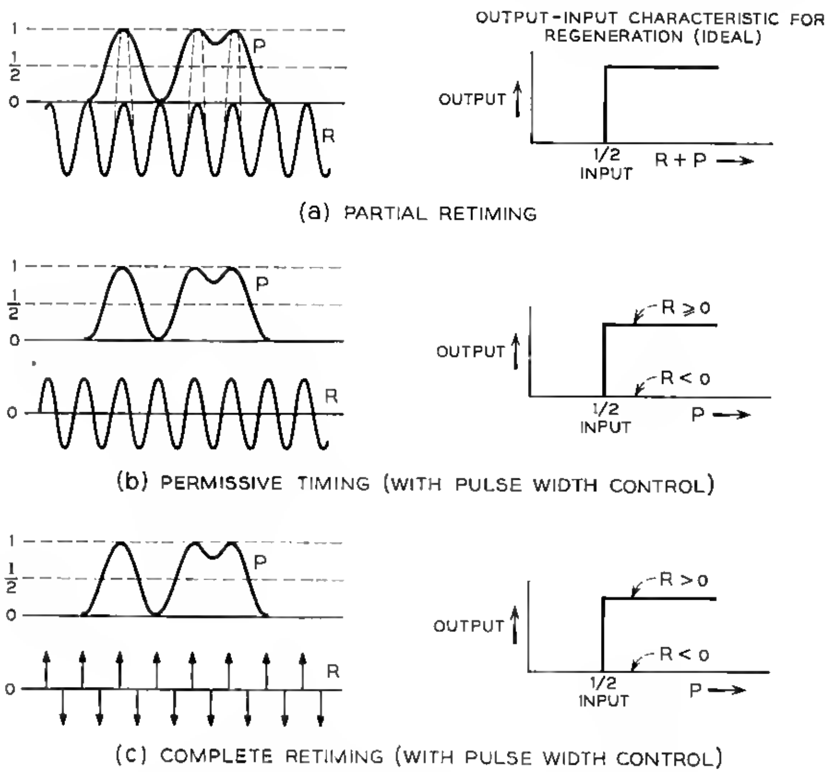

Fig. 2 — Retiming methods: (a) partial retiming, (b) permissive timing (with pulse width control), (c) complete retiming (with pulse width control).

may be obtained from either the incoming or regenerated pulse train or from a separate pair.

Another method for using the timing wave is illustrated in Fig. 2(b). We call this permissive timing with pulse width control. Both the incoming pulse train and the clock are put into an "and" gate. When the signal pulse exceeds the threshold and the clock is in its positive half cycle, standard current is fed to a regenerator (a blocking oscillator, for example) and the regeneration process is started. When the clock wave enters its negative half cycle, current is extracted from the regenerator to turn it off. In this manner the pulse width is constrained to be less than one-half cycle of the clock.

One final approach is depicted in Fig. 2(c). This approach is known as complete retiming with pulse width control. In this method, narrow pulses are generated at the positive-going or negative-going zero crossings of the timing wave and used for gating the incoming pulse train. Similarly, pulses generated at the negative-going or positive-going zero crossings are used to turn off the regenerator to control the width of the regenerated pulses. In passing we note that the narrower the sampling pulse, the smaller the fraction of time that interference is effective in obscuring a pulse decision. The importance of the width of the sampling pulse in combating crosstalk is discussed in the paper by Mayo.[3]

As discussed by Sunde[6] and Rowe,[10] complete retiming precludes timing from the reconstructed pulse train. Backward-acting timing is also taboo with permissive timing.

2.3 *Timing Wave Extractors*

In self-timed repeaters, several circuit configurations have been investigated for timing-wave extraction. Since the timing recovery problem is similar to that involved in synch recovery in TV, it is to be expected that circuit approaches investigated for that application should be pertinent here.[8] They include:

1. Tuned circuits
   a. single tuned — LC
   b. double tuned — LC and other narrow-band filters
   c. crystal filters
2. Controlled passive filter
3. Clutched (locked) oscillator
4. Phase-locked oscillator — APC loop
5. Phase- and frequency-locked oscillator — dual mode.

By far the simplest and cheapest circuit is the single tuned LC tank.

Most of the other possible implementations have been examined for this application.

Detailed discussion of the merits of each approach is beyond the scope of this paper. It suffices to point out that an automatic phase-control (APC) loop is the ideal timing wave extractor, and an APC loop for this application has been constructed. This relatively complicated circuit is available, if required, to mop-up timing deviations that propagate through a chain of repeaters with simple LC tanks. For the short repeater chains encountered in the exchange cable application, the LC tank with a loaded Q of 100 can be designed to meet jitter requirements, system economics, and space requirements. Therefore, in the remainder of the paper we will concentrate on this simple implementation.

2.4 *Sources of Timing Jitter*

In self-timed reconstructive repeaters using LC tanks for timing recovery, several sources of timing jitter and mistiming arise. They are (1) thermal and impulse noise, (2) mistuning, (3) finite pulse width, (4) amplitude-to-phase conversion in nonlinear devices, and (5) crosstalk.

2.4.1 *Thermal and Impulse Noise*

De Lange and Pustelnyk[9] have shown that thermal noise is not a serious contributor to timing jitter in the timing channel of a reconstructive repeater. Stated another way, if thermal noise is small enough such that the pulses can be recognized with low probability of error with no noise in the timing channel, then the addition of noise in the timing channel results in a negligible increase in the error probability. Physically this is to be expected since the narrow-band timing extractor accepts only a small fraction of the noise power for circuit Q's of the order of 100. This conclusion is equally valid for impulse noise for the same reason as above and also because the average rate of occurrence of impulses is considerably less than the bit rate or the minimum pulse density allowable in the system.

At this juncture it is appropriate to point out that the timing wave amplitude must exceed a certain minimum value to operate the timing gate and to limit some of the sources of mistiming to be discussed below. This presents a limitation on the minimum pulse density that can successfully be reconstructed in a repeater string for the transmission schemes of type 1a in Table I. It should be emphasized that this limita-

tion will prevail with any of the circuit implementations in a practical environment that includes timing recovery with a mistuned extractor excited by a baseband pulse train that permits spaces. Furthermore, this lower bound is a function of the statistics of the signal being transmitted, the allowable error rate, the kind of interference, the effective Q of the timing circuit, and the amplification following the timing tank. Once the timing wave has reached a sufficiently large amplitude, the higher the Q, the longer the gap between pulses that can be bridged. However, the higher the Q, the greater the phase slope of the tuned circuit and the smaller the allowable mistuning for a given signal-to-interference ratio. This inherent compromise between Q and mistuning will be discussed quantitatively below. In the exchange carrier application, it is necessary to limit the maximum number of spaces between pulses to about 15. This is achieved by eliminating the all-zeros code in the encoder. Attendant to this constraint is a small change in terminal distortion due to clipping.

2.4.2 *Mistuning*

When the tuned circuit is mistuned from the p.r.f. (pulse repetition frequency), the information-bearing pulses are sampled away from their peaks. This lowers the interference allowable for a specified error rate. As noted above, the higher the Q, the larger the shift in the zero-crossings of the timing wave at the output of the tuned circuit for a fixed mistuning. However, a high Q is desirable to bridge gaps and reduce timing jitter due to finite pulse width, as discussed below. Furthermore, the effect of mistuning is dependent upon whether forward- or backward-acting timing extraction is used, as well as whether partial or complete retiming is employed. We will shortly compare all of these techniques quantitatively.

2.4.3 *Finite Pulse Width and Pattern Effects*

As shown by Rowe,[10] when the pulses exciting the tuned circuit are not impulses or 50 per cent duty cycle rectangular pulses, the zero-crossings at the output of the tuned circuit are perturbed from their nominal positions. These deviations are dependent upon the pulse density (or pattern), the Q of the tuned circuit, and the shape of the pulses exciting the tank; and they differ for positive- and negative-going zero-crossings. This is a form of amplitude-to-phase conversion analogous to the amplitude-to-phase conversion in FM limiters.

Pattern jitter also occurs in partial retiming due to the above effects.

While backward-acting timing with ideal 50 per cent duty cycle rectangular pulses exciting the tuned circuit eliminates the finite pulse width effect, it does suffer from severe pattern jitter due to variation in timing wave amplitude with pulse density.

2.4.4 *Nonlinear Amplitude-to-Phase Conversion*

Any of the nonlinear circuits either preceding or following the tuned circuit will inevitably perturb the zero-crossings of the timing wave. In addition the regenerator, nominally a blocking oscillator, will not be an ideal zero-memory nonlinear device. Consequently, the spacing between reconstructed pulses will be altered by its inherent storage and will be dependent on the past history of the reconstructed pulse train.

2.4.5 *Crosstalk*

Intra- and inter-system crosstalk into the timing channel of a forward-acting repeater, either partially or completely retimed, results in a shift of the zero-crossings of the timing wave. We will devote the bulk of the paper to this problem.

2.5 *Preliminary Comparisons*

At this point, we will make preliminary quantitative comparisons of some of the methods of extracting and using the timing wave in a self-timed repeater. Our attention will be confined to the effects of mistuning and pattern jitter.

2.5.1 *Mistuning*

As noted previously, when the tuned circuit is mistuned from the pulse repetition frequency, tolerance to interference is reduced. To compare the various self-timing schemes quantitatively, we will make the following assumptions:

1. The information-bearing pulses are raised cosine pulses of base width, either T, $1.5T$, or $2T$* wide.

2. The output of the tuned circuit is a sinusoid whose peak-to-peak amplitude is equal to the height of the information-bearing pulse. For partial retiming, the positive peak is clamped to ground and added to the signal.

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* In the $2T$ case, the discrete component at the bit rate disappears. Therefore, nonlinear methods must be used to create a discrete line at the bit frequency for purposes of timing.

Under these conditions, we can use the methods developed by Sunde[6] to compare backward and forward-acting partial retiming with complete retiming. In the latter case, we assume that the phase shift in the zero crossings of the timing wave is given by its average value, approximately $2Q(\Delta f/f)$; $\Delta f/f$ is the fractional mistuning of the tuned circuit. Fig. 3 shows the reduction in allowable interference in the presence of mistuning for the three self-timing methods of interest. As expected, forward-acting partial retiming and complete retiming are far superior to backward-acting partial retiming. This is particularly noticeable for the case where the pulse width is from 1.5 to 2 time slots wide at its base. It will be shown that a pulse shape in this region is required to minimize the effects of crosstalk interference in making the pulse-no pulse decision. Based on the fact that there are several sources of impairment that must share the allowable margin against error, that portion of the margin allocated to mistuning must be as small as possible, consistent with presently available components. In the present state of the art, including initial misplacement and aging, it appears that the maximum phase shift in the tuned circuit (LC tank) can be held to about $\pm 30°$ for a Q of about 100. This corresponds to $2Q(\Delta f/f) \doteq 0.6$, or $\Delta f/f = 0.003$. From Fig. 3 and the fact that equalization to minimize crosstalk interference must yield a pulse close to the 1.5 to $2T$ cases, it can be seen that backward-acting timing should not be used for this application.

It should be pointed out that there is nothing restrictive about the use of raised cosine pulses in making this point. It can be shown that the conclusion arrived at above remains valid for other similar pulse shapes, time limited or not, and for other practical ratios of timing wave amplitude to pulse peak. Indeed, for larger ratios of timing wave amplitude to pulse peak, the stability problem is further aggravated. This is due to the positive feedback nature of the timing loop in backward-acting timing. A straightforward extension of Sunde's work will verify this contention. In addition, experimental work by A. C. Norwine* confirms this expected behavior.

2.5.2 *Finite Pulse Width and Pattern Effects*

In this category, we will compare partial retiming (forward-acting) with complete retiming for both periodic and random pulse patterns. With periodic patterns, both raised cosine and Gaussian pulses will be considered. Several ratios of average timing wave amplitude to peak

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* Private communication.

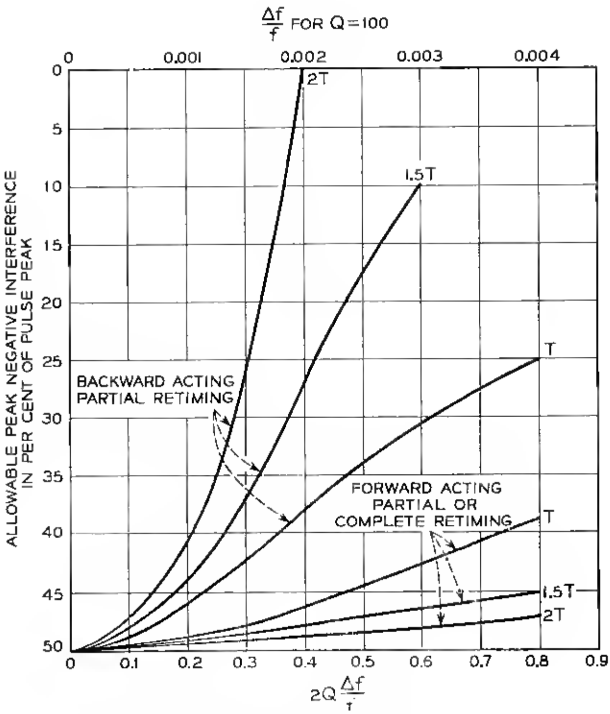

Fig. 3 — Effects of mistuning.

pulse height will be considered with partial retiming. We continue to consider binary (1,0) pulse trains.

2.5.2.1 *Periodic Pulse Patterns — Partial Retiming.* To divorce pattern effects from mistuning, we assume that the peak of the timing wave occurs at the pulse peak. Further, the positive peak of the timing wave is pinned to ground and the timing wave amplitude is given by

$$-\frac{an}{2M}\left(1 - \cos\frac{2\pi t}{T}\right) \tag{1}$$

where

a is the peak-to-peak amplitude of the wave when all pulses are present; i.e., $n = 1 = M$, and

n is the number of pulses that occur in an M-bit word.

When $n \neq 1$, we assume that the pulses are adjacent, and that the time slot examined is the one containing the last pulse in the group. In the following, we will specialize to $n = 1$, and M will vary from 1 to 8.

The assumptions underlying (1) are, first, that the pulses exciting the tuned circuit are very narrow pulses obtained by processing the incoming signal, and second, that the timing wave amplitude in a real repeater is dependent upon the density of pulses exciting the tuned circuit. While the first assumption does not correspond to the practical case, it does give an optimistic view for partial retiming. We will find that even with this idealization, partial retiming is considerably inferior to complete retiming in maintaining the proper pulse spacing under both steady-state and random conditions. It is possible to drop the first assumption here, as we do for complete retiming. The additional analysis simply lends further emphasis to the conclusion.

Two pulse shapes will be considered. First, the raised cosine with

$$P(t) = \frac{1}{2}\left(1 + \cos\frac{2\pi st}{T}\right) \quad \text{where} \quad |t| < \frac{T}{2s} \qquad (2)$$
$$= 0 \qquad \text{elsewhere.}$$

The pulse width is T/s. We consider $s = 1, \frac{2}{3}$. Secondly,

$$P(t) = e^{-(\pi^2/\ln 2)(f_6 t)^2}. \qquad (3)$$

In (3), f_6 is the frequency at which the transform of the Gaussian pulse is 6 db down from its low-frequency asymptote. Alternatively, we can write the Gaussian pulse as

$$P(t) = e^{-4\ln 10(t/T_w)^2}. \qquad (4)$$

In (4), T_w is the pulse width between points where the pulse amplitude is 0.1 of its peak.

In partial retiming, the incoming pulse is regenerated at the instant when the sum of the signal plus the timing wave exceeds the slicing level, which is assumed to be at $\frac{1}{2}$. The resulting problem can be solved graphically, iteratively, or, for some cases, analytically. Fig. 4 displays the results of these computations for raised cosine pulses.* The ordinate gives the steady-state phase shift in degrees, measured from the pulse peak, as a function of the pulse pattern. In all cases we have subtracted out the phase shift corresponding to all pulses present. The important point is the fact that the difference in phase shift between all pulses present and $\frac{1}{8}$ present is quite large: about 60° for the widest pulse and 50° for a pulse width of $1.5T$. Fig. 5 presents the same information for Gaussian pulses, and the results are not substantially different from those obtained for raised cosine pulses. It is worth noting that inter-

* Similar computations have been made in an unpublished memorandum by W. M. Goodall and O. E. De Lange.

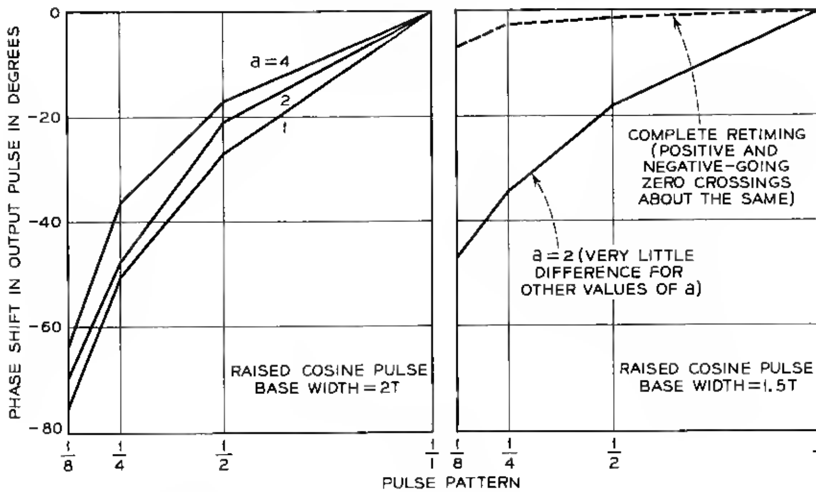

Fig. 4 — Pattern effects — raised cosine pulse: (a) base width = $2T$, (b) base width = $1.5T$.

ference riding on the leading edge of the pulse will result in additional timing jitter.

2.5.2.2 *Periodic Pulse Patterns — Complete Retiming.* In complete retiming, timing jitter results from the finite width of the pulses exciting the tuned circuit and the variation of pulse density. Using a minor mod-

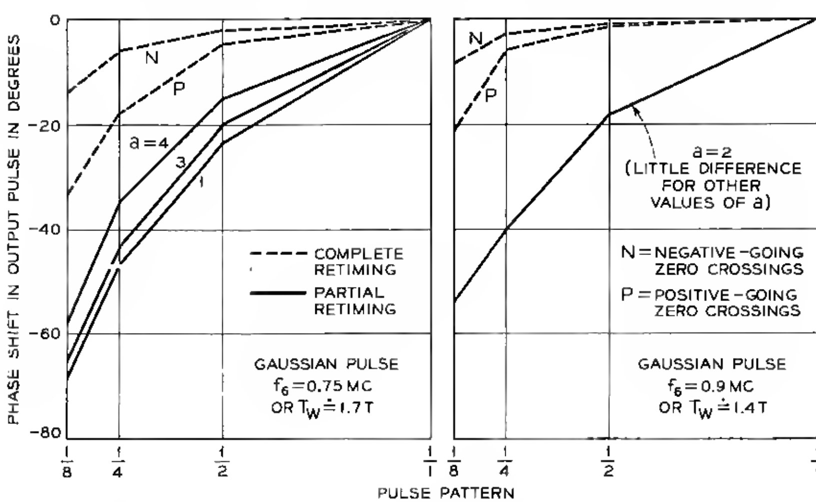

Fig. 5 — Pattern effects — Gaussian pulse: (a) $f_6 = 0.75$ mc, (b) $f_6 = 0.90$ mc.

ification of Rowe's method, we have displayed the phase shift in the zero crossings at the output of the tuned circuit in Figs. 4 and 5. A Q of 100 is assumed, and both positive- and negative-going zero-crossings are considered. It can be seen that the phase shift in this case is considerably smaller than in the case of partial retiming. Since these figures display static performance, we cannot immediately conclude that the resulting distortion in the reconstructed analog signal will be either tolerable or intolerable. Distortion at the output of the demultiplex filter in the terminal due to phase modulation on the PAM samples is dependent upon both the amplitude of the jitter and its rate of change. These quantities in turn depend upon the bandwidth (Q) of the timing extractor used in each repeater. Without going into detail, based on the dynamic effects of timing jitter propagation due to pattern shift in a string of repeaters, it can be concluded that partial retiming will be unsatisfactory for this application. Furthermore, the additional circuitry required in complete retiming over that needed for partial retiming (forward) is relatively simple and cheap. Finally, and most important, complete retiming makes pulse width control relatively easy, and the use of narrow sampling pulses yields a substantial crosstalk advantage.[3]

From Fig. 3, we see that wide pulses are desirable to minimize the effects of mistuning, while Fig. 5 shows that narrower pulses are desirable for minimizing jitter in the zero-crossings at the output of the timing wave extractor. Furthermore the positive-going zero-crossings are perturbed to a larger extent than negative-going zero-crossings for sparse patterns. This suggests separate equalization in the timing path to narrow the pulses prior to excitation of the tuned circuit. Slicing in the timing path is useful in this regard and also serves to eliminate low-level interference in the absence of a pulse. This will be emphasized later in connection with crosstalk.

2.5.2.3 *Random Pulse Patterns.* Another view of the dynamic effects of pattern jitter may be obtained by determining the probability distribution of phase jitter for random pulse patterns. This approach will be published in another paper[17] where we will include both mistuning and finite pulse width effects. This information can be used to further cement our choice of complete retiming for this application.

2.6 *Interim Summary*

At this point a short summary of our preliminary conclusions is in order. Based primarily on mistuning and pattern effects, with a peek ahead at crosstalk considerations, it is concluded that complete retiming with pulse width control should be used for a self-timed repeater.

This follows from the fact that our objective is to leave most of the margin against error to crosstalk interference in order to avoid pair selection. For this same reason, and the fact that it is readily approached, complete regeneration should be employed.

It should be apparent that pattern effects ideally can be eliminated with timing wave added or by transmitting the timing wave on a separate pair. There are important economic and technical reasons why these approaches are undesirable and we will cover them in Section V.

III. NATURE OF NEAR-END CROSSTALK

3.1 *General*

With the above preliminaries largely disposed of, we can concentrate on the crosstalk problem. This problem arises when we consider both directions of transmission for a single 24-channel system and is further compounded when many 24-channel systems are transmitted on separate pairs in the same cable bundle. In effect, such a system is a combination of time division and space division, and a typical repeater-to-repeater link can be depicted in block diagram form as shown in Fig. 6.

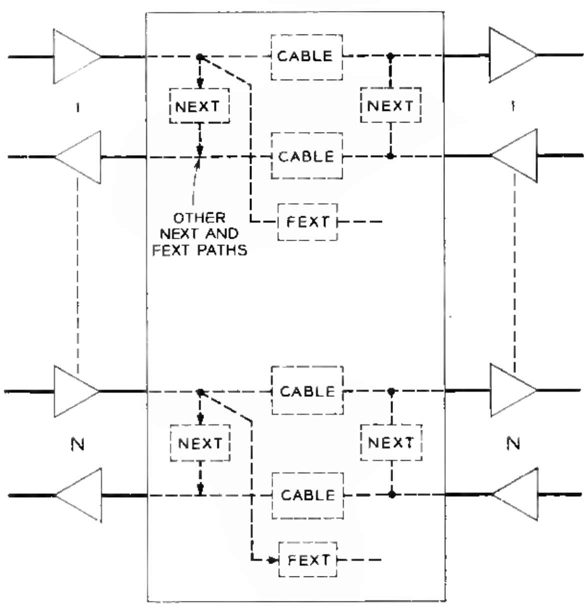

Fig. 6 — "Channel" for N PCM systems.

We could write a matrix relationship between the transforms of the voltages at the $2N$ output terminals and the $2N$ input terminals. The elements of the matrix would contain the transfer function of the direct cable paths on the main diagonal, and the transfer functions of the near-end and far-end crosstalk paths would reside in the off-diagonal terms. Such a representation, though elegant, would be of little use without a detailed characterization of the elements of the matrix (channel). At the present state of our knowledge, the above viewpoint is not too useful.

The most important couplings fall in the category of near-end crosstalk coupling (NEXT) between pairs operating in different directions of transmission. A comprehensive treatment of the physical mechanism from which crosstalk arises will not be attempted. A useful image of the situation results from considering the $2N$ pairs (for N systems) as $2N$ lossy coupled, tapped delay lines in which the "taps" (inductive and capacitive coupling due to unbalances) have both random amplitude and spacing. For our purposes, the macroscopic view obtained from looking at the $4N$ ports of this network will suffice. Furthermore, it will be convenient to make a further abstraction from reality and define an "equivalent crosstalker" and an "equivalent crosstalk path" to replace the complicated statistical model given above. This approach will permit us to attach a "crosstalk figure of merit" to each transmission scheme, which will serve as a valuable aid in sorting out those approaches most tolerant to near-end crosstalk (NEXT).

3.2 *NEXT Measurements*

3.2.1 *Single-Frequency Distributions*

Over the past twenty years or so, hundreds of thousands of single-frequency measurements have been made of crosstalk between pairs in trunk cables. Relatively few of these measurements have been made in the 100-kc to 10-mc region. A small number of measurements in this frequency range have been made at Bell Laboratories by B. Smith on a unit-constructed cable. We have used these preliminary data as a guide-post to choose a transmission scheme.* The distribution of these measurements at 1.5 mc (close to the bit rate of interest) is shown in Fig. 7. Two cases of interest are shown. The worst case prevails when the interfering pair is in the same unit as the pair into which it is crosstalking. When the two pairs are in adjacent units, crosstalk coupling is reduced.

* It is to be emphasized that these are preliminary data and are included to convey a feel for the magnitude of the problem; they do not necessarily typify the average cable plant.

The most favorable situation (not shown) occurs where opposite directions of transmission are located in diametrically opposed units of the cable. Where unit integrity is maintained throughout the length of the cable, it is obviously prudent to take advantage of the lowest crosstalk coupling in installation. However, unit integrity is not universally adhered to. Therefore, for purposes of discussion we will use the data on Fig. 7 corresponding to the worst case.

It should be noted that, as shown, the distribution of pair-to-pair crosstalk is distributed according to a log normal distribution. There is some theoretical justification for this shape over most of the crosstalk loss range. It is to be expected from physical considerations that the distribution should truncate in the neighborhood of the tails. Indeed, R. J. Herman in work at Bell Laboratories has shown that the log normal hypothesis is not supported by the data in this region.

3.2.2 *Loss vs Frequency*

Single-frequency crosstalk loss measurements are most useful for comparing some transmission schemes with respect to crosstalk in the

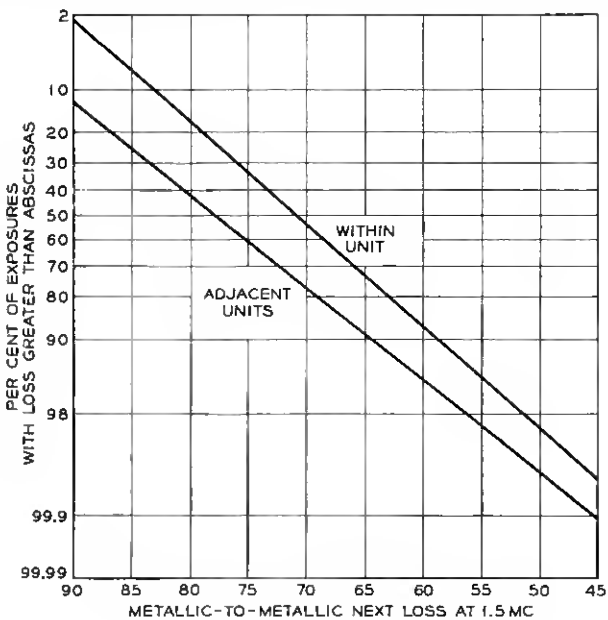

Fig. 7 — Preliminary near-end crosstalk loss distribution.

timing channel. However, these data are insufficient to define the effect of crosstalk interference on the desired information-bearing signals. An appraisal of the effect of crosstalk on the desired pulse train requires the determination of the loss and phase of the crosstalk path or, alternatively, the pulse response of this path. Since the interfering signal is made up of a combination of random pulse trains, and since the paths they traverse to the system being interfered with also come from a random family, a complete statistical description of the situation is extremely complicated and nonexistent. Measurements of the NEXT loss between pairs, as a function of frequency, display a broad structure in which the loss decreases with increasing frequency at about 4.5 db per octave (coupling proportional to $f^{3/4}$) for frequencies greater than about 200 kc. This is in good agreement with some unpublished theoretical work of D. K. Gannett of Bell Laboratories. In addition to this nominal smooth behavior as a function of frequency, measurements have revealed other more rapid variations with frequency. This fine structure may be attributed to relatively large localized capacitive and inductive unbalances. These results are in accord with the model specified previously as a visual aid and give a qualitative explanation of the classes of pulse responses observed with pulse excitation of the crosstalk path.

3.2.3 *Simplification*

In our comparison of various transmission schemes, we will find it convenient to neglect the fine structure associated with the crosstalk path. We take this bold step with the obvious realization that we preclude a completely definitive evaluation. It was mandatory to make this abstraction from reality early in the system development when detailed crosstalk data were not available and a development decision had to be made. The principal advantage of this simplification is that it permits us to isolate the classes of codes most tolerant to NEXT interference in the timing channel of the disturbed repeater.

It was convenient and expedient to go one step further and replace the multiplicity of crosstalk paths and interferers by a single capacitive coupling and a single interferer for purposes of repeater design and experimentation. An equivalence between this gross simplification and the real world can be determined empirically. Mapping of results obtained in the simple domain onto the real world by analytical means is an extremely difficult chore. We will have a little more to say about this correlation of models in a later section.

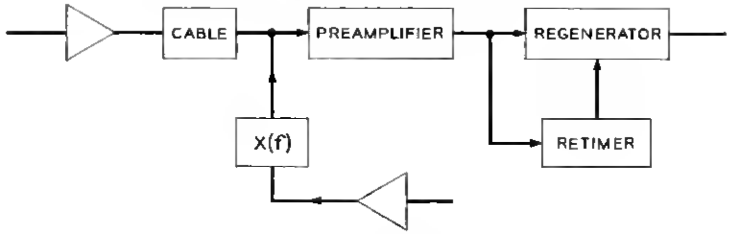

Fig. 8 — "Equivalent" crosstalk situation.

IV. CROSSTALK INTO THE TIMING CHANNEL

4.1 *Assumptions*

With the simple "smooth" crosstalk model we can begin to be more quantitative in our definition of the crosstalk problem, particularly with respect to timing. For purposes of definition, and later comparisons, we will consider the situation depicted in Fig. 8. The following assumptions are appropriate:

1. Nominal repeater spacing is 6000 feet on 22-gauge cable, and "within-unit" distribution of metallic-to-metallic NEXT is applicable.

The line loss corresponding to this length at 55°F is shown on Fig. 9.

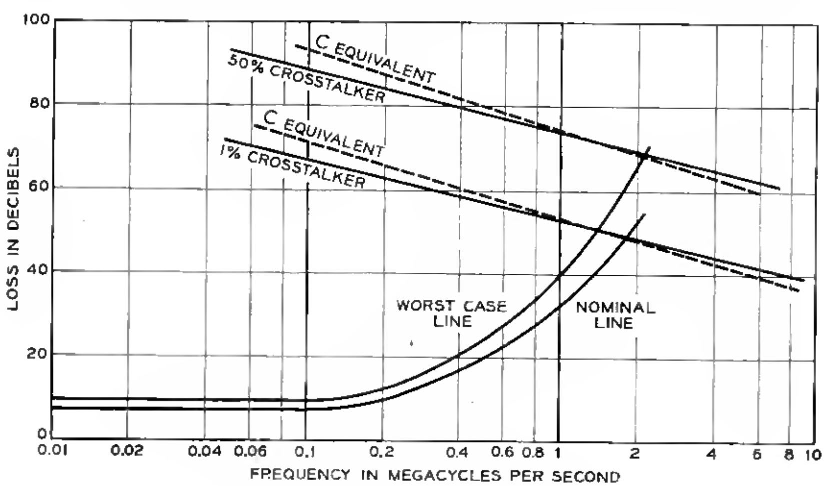

Fig. 9 — Line loss and crosstalk loss.

In addition, the line loss under extreme temperature conditions (100°F), under extreme manufacturing deviations (3σ limit), and line length of 6500 feet is also depicted. The mean crosstalk and the 1 per cent crosstalk loss are also shown using both 6 db and 4.5 db per octave extrapolations from the mean and 1 per cent values of Fig. 7.

2. A disparity of pulse densities of 7/1 on disturbing and disturbed pairs is quite likely.

This assumptiou is of course intimately tied up with the code pattern transmitted by the terminals and can arise when there are several idle channels in the disturbing system and the coder is misbiased. Under conditions of misbiasing, the coder may put out level 63 for the idle circuit or no speech condition instead of 64. Clearly this also depends upon noise riding on the reference. There are several techniques for minimizing the effects of this disparity in pulse densities on repeater timing, and they will be considered anon. An 8/1 disparity in pulse densities can occur for speech transmission but with a low probability. For transmission of high speed data over T-1 lines, the disparity in pulse densities can be 8/1 with significant probability.

3. Finally, we will determine tbe allowable crosstalk at the pulse repetition frequency such that the desired timing frequency component at the input to the tuned circuit is 6 db stronger than the timing frequency component that traverses the crosstalk path. This corresponds to a maximum of 30° phase shift in the timing signal relative to the position of the desired pulse peak. It turns out that this amount of jitter is tolerable as far as pattern jitter effects on the reconstructed PAM signal are concerned for the handwidths ($Q = 100$) of the tuned circuits used in the repeaters.[3] We will not be concerned with these dynamic effects in any detail. Our picture will, by the large, be a static one, and the bandwidth of the tuned circuit will not enter. The allowable crosstalk loss as determined above is defined as the "crosstalk figure of merit" for a transmission scheme. The smaller this crosstalk figure, the larger the number of pairs that can be used for PCM transmission.

From tbe data on Fig. 9, we can anticipate the difficulty associated witb transmitting timing information down the repeatered line at the bit frequency. For example, if we assume the worst case line loss and equal transmitted timing components on both disturbing and disturbed pairs, then $53 + 6 = 59$ db is the required crosstalk loss to meet the 30° phase shift requirement. With this crosstalk figure of merit, it is apparent from the distribution of Fig. 7 that pair selection cannot be ruled out even with a single interferer and no adverse disparity in pulse densities.

It should be quite evident from an extrapolation of Fig. 9 that 12,000-foot spacing, twice the normal load-coil spacing, is an unworkable situation.

4.2 *Pair Selection*

Now that we have raised the specter of pair selection, it will be instructive to bring the magnitude and scope of the associated measurement problem to the fore. Fig. 6 gives a picture of the crosstalk situation at the extremities of a repeater-to-repeater link. For N PCM systems in the cable, there are N^2 crosstalk exposures at each end of the link. In an L-mile route containing N PCM systems, approximately $2N^2L$ exposures exist. As an example, consider the installation of 20 PCM systems, each 25 miles long. Approximately 20,000 exposures exist, and the measurement, characterization and exclusion of pairs is an enormous and expensive task. To avoid this problem we must choose a transmission method with a high degree of tolerance to intra- and inter-system crosstalk. More precisely, a transmission scheme sufficiently insensitive to near-end crosstalk interference must be chosen such that the probability of failure of a single repeater (due to this cause), out of the approximately $2NL$ installed, is small. This calls for a low crosstalk figure of merit.

4.3 *Unipolar Pulse Train*

The simplest and most common form for transmitting binary PCM is to represent a one by a pulse and a zero by the absence of a pulse. Under these conditions, with pulses and spaces uncorrelated, it is well known that the power spectral density of the pulse train is

$$P_1(f) = \frac{|\,G(f)\,|^2\,p(1-p)}{T} + |\,G(f)\,|^2\frac{p^2}{T^2}\sum_{n=-\infty}^{\infty}\delta(f-nf_r). \quad (5)$$

In (5), p and $(1-p)$ are the probabilities of pulse and no-pulse respectively, $G(f)$ is the Fourier transform of the pulse shape, and T is the reciprocal of the pulse repetition frequency f_r. The presence of a discrete line in the spectrum at the bit rate (assuming $|\,G(f_r)\,| \neq 0$) permits the extraction of timing information from the pulse train by means of a simple tuned circuit. With a random pulse train on both disturbing and disturbed pairs, the crosstalk figure of merit for worst case line loss is 59 db as before. Even with nominal line loss the figure of merit is 49 db. In either case, pair selection cannot be excluded for only one system. Unfavorable disparity in pulse densities of 7/1 raises the worst case figure

of merit by 17 db to 76 db. From Fig. 7 we note that only 40 per cent of the exposures have crosstalk loss exceeding 76 db. This shows clearly that this simple transmission scheme is not suited to the exchange plant environment. Obviously, the direction to proceed is to consider transmission schemes in which the timing information is transmitted at a frequency below the bit rate where the line loss is reduced and the crosstalk loss is increased. In the succeeding sections we will present some methods for achieving this end.

V. OTHER PULSE TRANSMISSION SCHEMES

5.1 *General*

All of the transmission schemes that we will consider have a common thread. They involve a conversion of the binary train to a three-level code (pseudo-ternary) for transmission over the line. Conversions are accomplished on a bit-by-bit basis. Translation from the binary code to a three-level code by operating on multiple bits or binary words will not be discussed. In addition, hybrid combinations (series or parallel or both) of the bit-by-bit converters are possible. However, the latter two approaches have been ruled out for this application, either on the basis of economics or the fact that they do not afford substantial technical advantages over the simpler methods. It should be understood, however, that the provision of a three-level reconstructive repeater permits the utlization of some of the more complex code translators at the terminal should unforeseen conditions dictate their choice.

Several of the pseudo-ternary pulse trains will be described prior to placing them in the crosstalk environment. In addition, other factors will be brought to bear in the comparison of the various three-level codes. Consideration of these other factors involves judgments of the degree of difficulty and the costs associated with the realization of the various approaches.

One of these additional factors involves low-frequency suppression. Transformer coupling is required to couple an unbalanced repeater to the balanced line and to provide a phantom path for remotely powering the repeaters. Transmission of a pulse through transformers results in a long transient undershoot that extends over several time slots and interferes with subsequent pulses in the train. This of course reduces tolerance to crosstalk. There are a host of techniques that have been used to combat this low-frequency wander. A summary of most of these methods is given in Ref. 11, and we do not intend to review them in this paper. We

have found that many of the circuit approaches for minimizing or theoretically eliminating the effects of low-frequency suppression that appear ideal on paper have been found wanting when actually implemented and placed in a real world environment. Pulse trains whose spectra contain discrete lines at dc suffer most from low-frequency suppression. This is another reason for discarding unipolar.

A second factor of interest involves compatability of PCM systems with AM carrier systems using pairs in the same cable. Compatability is a function of the number of PCM systems involved, the crosstalk loss in the frequency region occupied by the AM system, and other details and layout of the particular AM system. An indication of the *relative* compatability of the various PCM transmission approaches due to interference from PCM into the AM system may be obtained from a comparison of their power spectra. Since crosstalk loss decreases with increasing frequency, the AM system that extends to the highest frequency will be most affected. N carrier fits this picture. Therefore, we will use a frequency close to the top of the N carrier band as a bench mark for comparison of the relevant power spectra. For convenience we choose $f_r/6 \doteq 257$ kc as the representative frequency. Since N carrier employs frequency frogging, other higher frequencies are also of importance at a high-low N repeater. In particular, frequencies up to 440 kc are of importance. This is one reason why timing at $f_r/4$ does not seem attractive.

5.2 *Time Polarity Control* (TPC)

The first transmission scheme we consider is called time polarity control. It was suggested and demonstrated by L. C. Thomas. In this approach, time slots are labeled alternately positive and negative. If a unipolar pulse occurs in a positively labeled time slot, it is transmitted unaltered. On the other hand, if a unipolar pulse occurs in a time slot with a negative label, the pulse is transmitted with negative polarity. Zeros in the unipolar train are unaffected. The block diagram of a circuit for achieving this end is shown in Fig. 10, along with an example of an idealized pulse train before and after conversion. Intuitively, discrete lines in the power spectral density of the TPC train are expected at multiples of half the bit rate. This inherent periodicity may be demonstrated by computing the ensemble average of the TPC train. To do this, we assume that the original binary pulse train has independent pulses and spaces that occur with probability p and $1 - p$. Furthermore, we assume that the positive and negative pulse shapes in the TPC train are given by $g_1(t)$ and $-g_2(t)$, respectively. The latter assumption is introduced to

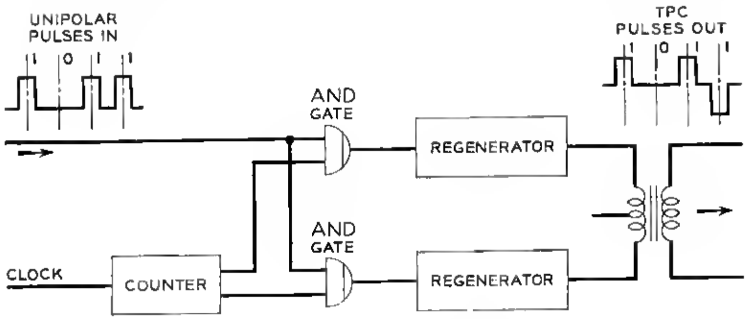

Fig. 10 — TPC converter.

account for practical differences in the two circuits that generate the positive and negative pulses. Under these conditions, the ensemble average of the pulse train is

$$\text{av } x(t) = \frac{p}{2} \sum_{n=-\infty}^{\infty} (g_1(t - 2nT) - g_2(t - (2n + 1)T)). \tag{6}$$

That (6) is periodic with period $2T$ can be seen from

$$\begin{aligned} \text{av } x(t + 2T) &= \frac{p}{2} \sum_{n=-\infty}^{\infty} (g_1(t - (2n - 2)T) - g_2(t - (2n - 1)T)) \\ &= \text{av } x(t) \end{aligned} \tag{7}$$

by making a change in the summation index from $n - 1$ to m.

The power spectral density for TPC may be computed by any one of several approaches† to give

$$P(f)_c = \frac{p(1 - p)}{2T} [| G_1 |^2 + | G_2 |^2] \tag{8}$$

for the continuous portion of the spectrum. The discrete spectrum is

$$P(f)_d = \frac{p^2}{4T^2} [| G_1 |^2 + | G_2 |^2 - 2ReG_1G_2^*e^{-j2\pi f/T}] \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{nf_r}{2}\right) \tag{9}$$

In the ideal case when $G_1 = G_2$

$$P_2(f) = \frac{p(1 - p)}{T} | G_1 |^2 + \frac{p^2}{T^2} | G_1 |^2 \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{(2n - 1)f_r}{2}\right). \tag{10}$$

From (10) it can be seen that the continuous part of the spectrum is

† Signal flow graph techniques[12,13] were used in deriving the expressions for the power spectral densities.

identical with that of unipolar assuming the same pulse shape in both cases. In ideal TPC discrete lines occur at odd integer multiples of one-half the bit rate. This permits timing at half the bit rate. Reconstruction of the original binary train at the receiver is achieved simply by rectification of the TPC train consisting of non-overlapping pulses. Half bit rate timing in the presence of a random crosstalk interferer with a crosstalk loss of 39 db at half the bit rate yields a crosstalk figure of merit of $39 - 4.5 = 34.5$ db for worst case line loss.

It is advisable for other comparisons to catalogue certain features of the power spectrum of TPC. Unlike unipolar, TPC with balanced pulses has no discrete line at dc. This eases but does not eliminate the low-frequency suppression problem. It still is necessary to consider long runs of alternating positive (or negative) pulses and spaces. Lack of dc transmission will increase the susceptibility of the system to errors due to crosstalk. DC restoration operating on both the positive and negative peaks can be employed to reduce this effect. This is a relatively difficult circuit problem and results in an increased repeater cost over other methods to be considered.

There are several modifications of the basic TPC converter that can be made to achieve other codes. Increasing the number of stages in the couuter produces higher-order TPC. For an M-stage counter; M time slots are labeled positive, the next M negative, and the cycle is repeated. If the positive and negative output pulses are identical (except of course for sign), the continuous spectrum of TPC-M, is identical with unipolar. Discrete lines appear at odd integer multiples of $f_r/2M$. For $M > 1$, the fundamental occurs at frequencies occupied by AM carrier systems and precludes compatability between PCM and the AM system. Furthermore, more instrumentation is required in the timing path of a repeater to process the signal preparatory to performing complete retiming with pulse width control.

5.3 *Bipolar*

Another and more useful modification of the converter of Fig. 10 results when the clock input to the counter stage is replaced by the incoming unipolar train, as in Fig. 11. The output pseudo-ternary code is thereby constrained such that two successive pulses, whenever they occur, must be of opposite sign. We assume the same conditions on the unipolar train used previously. Further, it is assumed that the transforms of the positive and negative pulses are G and $[-(1 + a)G + G_1]$ respectively. This brings out the differences in the two shapes more explicitly.

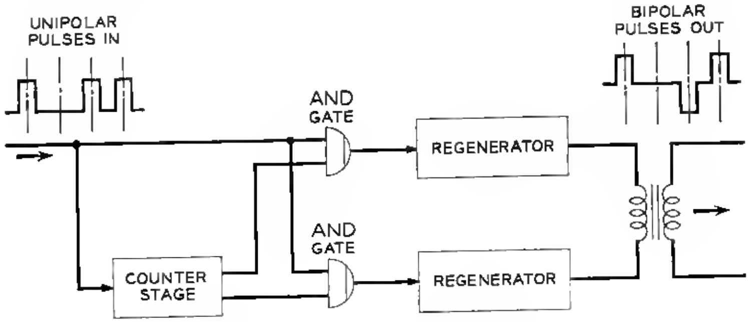

Fig. 11 — Bipolar converter.

In the ideal case (balanced pulses) $a = 0 = G_1$. With the general assumptions, the resulting expression for the continuous part of the power spectrum is long and will be covered elsewhere. The discrete spectrum is given in general by:

$$P(f)_d = \frac{p^2}{4T^2} \mid -aG + G_1 \mid^2 \sum_{n=-\infty}^{\infty} \delta(f - nf_r). \tag{11}$$

This obviously disappears under ideal conditions. Subject to this idealization, the spectrum contains only the continuous component given by:

$$P_3(f) = \frac{p(1-p)}{T} \mid G \mid^2 2 \left[\frac{1 - \cos \omega T}{1 + 2(2p-1) \cos \omega T + (2p-1)^2} \right]. \tag{12}$$

For the same pulse shapes in unipolar and bipolar

$$P_3(f) = 2 \left[\frac{1 - \cos \omega T}{1 + 2(2p-1) \cos \omega T + (2p-1)^2} \right] P_1(f)_c . \tag{13}$$

The subscript on P_1 indicates the continuous component of (5). With 50 per cent duty cycle rectangular pulses, the spectrum is shown in Fig. 12 (labeled $N = 1$) for $p = 1/2$. From this figure or (13) it is apparent that the spectrum has nulls at integer multiples of the bit rate (as well as nulls where $\mid G \mid = 0$). Absence of power at dc and the reduction of the spectrum in the neighborhood thereof eases requirements on the low-frequency performance of the transformers. This also follows from the fact that no two successive pulses, independent of their spacing, can be of the same sign.

In making this conversion we have suppressed all discrete components that might be useful for timing. As shown in the Appendix, rectification of the bipolar train produces discrete lines in the spectrum, and the com-

ponent at the bit rate can be used for purposes of timing.* The question arises as to where the energy for timing comes from. The analysis given in the Appendix (Section A.1) shows that the timing component arises *largely* from the region where the bipolar power spectrum is most concentrated. This is in the neighborhood of half the bit rate, and we will use this frequency to characterize the "timing frequency" for this method of transmission.† This result is intuitively satisfying since the rectifier acts as a frequency doubler. A square law rectifier has been assumed in the derivation given in the Appendix for several reasons. First, the square law device serves to reduce low-level interference in the absence of pulses. In this way, the adverse pulse density effects are reduced. Secondly, the square law assumption is a good approximation to a combination of slicer and rectifier that is actually used in the physical embodiment of this scheme. Finally, the square law characteristic is somewhat more convenient for analysis than the actual symmetrically biased rectifier.

The reader familiar with the literature will recognize that in the case $p = 1/2$, the spectrum of bipolar is identical with Meacham's twinned binary.[14] This follows from the fact that in Meacham's approach, twinned binary is generated by taking the unipolar train, delaying it by one time slot, and subtracting it from the original. This modifies the spectrum of unipolar by $| 1 - e^{-j\omega T} |^2 = 2(1 - \cos \omega T)$, which is identical to the modifying factor of (13) for $p = 1/2$. Despite the fact that the spectra are identical for equally likely unipolar pulses and spaces, the relationships between the original unipolar code and the transmitted code differ for the two methods. If the probability of error (both insertion and deletion) for the unipolar train is P_e for interference that is equally likely to be positive and negative, then it can be shown that the error probabilities for bipolar and twinned binary are $(3/2)P_e$ and $2P_e$ respectively. The above figures assumed that the repeaters to reconstruct either train do not have the bipolar constraint built in. In addition, due to the manner in which the twinned binary is converted to binary at the receiver, double errors per word are much more likely than in bipolar. For purposes of classification, bipolar belongs to the class of codes in which the conversion from unipolar is digital and the reconversion is analogue. In twinned binary the reverse is true.

There are two points of departure from the bipolar converter of Fig.

* A timing component could be added at the spectral null at the bit rate. This would not help the crosstalk timing problem except for the adverse pulse density effect.

† Obviously energy for timing does not come from a single frequency. However the characterization is convenient. Further clarification of this point is given in the Appendix and Section VI.

11. Increasing the number of counter stages to N results in a code in which N successive unipolar pulses are transmitted positive and the next N, wherever they occur, are transmitted negative. Spaces are unaffected. We have called such a pseudo-ternary code N-pulse. For $N = 2$, $p = 1/2$, and balanced pulses, the power spectrum has a continouus component only, given by:

$$P(f) = \frac{4(1 - \cos \omega T)(3 - 2 \cos \omega T)}{5 - 12 \cos \omega T + 8 \cos^2 \omega T} P_1(f)_c. \tag{14}$$

The spectrum is shown in Fig. 12 for 50 per cent duty cycle rectangular pulses. The power spectrum under similar conditions for $N = 3$ is also shown in this figure. As before, rectification can be used to convert back to unipolar.

In N-pulse, for dense pulse patterns the low-frequency suppression problem is more severe than in straight bipolar. Furthermore, the spectrum peaks up at low frequencies, thereby increasing the interference sprayed into AM systems using pairs in the same cable. Therefore, for this application, N-pulse is inferior to straight bipolar.

5.4 *Higher-Order Bipolar*

Another possible direction, first suggested by M. Karnaugh, involves paralleling bipolar converters and routing the unipolar pulses to each of

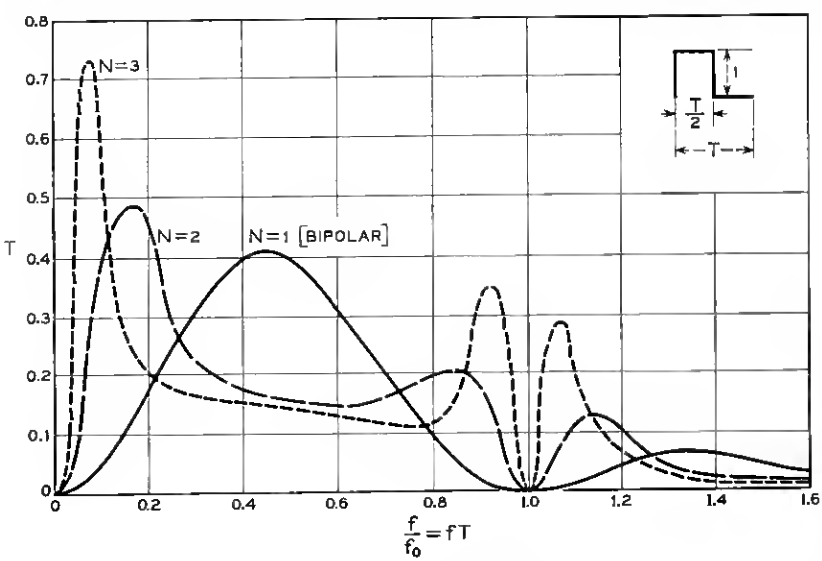

Fig. 12 — N pulse power spectra.

the converters in sequence. Such a composite converter is shown in Fig. 13. We will only give results for the power spectrum for balanced pulses and $p = 1/2$. With these conditions, for N converters in parallel the power spectral density is

$$P(f) = 2[1 - \cos N\omega T]\, P_1(f)_c . \tag{15}$$

Rectification suffices to reconstruct the original binary train.

The most interesting realization for our purposes is interleaved bipolar i.e., $N = 2$. It can be seen from (15) that the spectrum contains nulls at dc and integer multiples of half the bit rate. This permits the addition of a sinusoidal component at half the bit frequency to the pulse train for purposes of timing. In this manner, timing jitter due to finite pulse width effects is ideally removed, as is the adverse pulse density penalty. These are the principal features of this approach. As in 2-pulse, two pulses in a row may be of the same sign, thereby roughly doubling the low-frequency tail in the succeeding time slot and reducing tolerance to cross-talk. The addition and extraction of the sinusoidal component from the pulse train is not an easy nor an inexpensive circuit problem, and in practice results in an inevitable interaction between the pulse train and the sinusoidal signal to impair the decision-making process in the repeater. Specifically, balance requirements on the positive and negative pulses are particularly severe to suppress the discrete component in the pulse spectrum. The added timing component can be made sufficiently large to overcome the unbalances. However this increases the power-handling requirements of the repeater. Complete retiming and pulse width control are more difficult to implement with this approach than in bipolar for 50 per cent duty cycle pulses. The continuous spectrum at

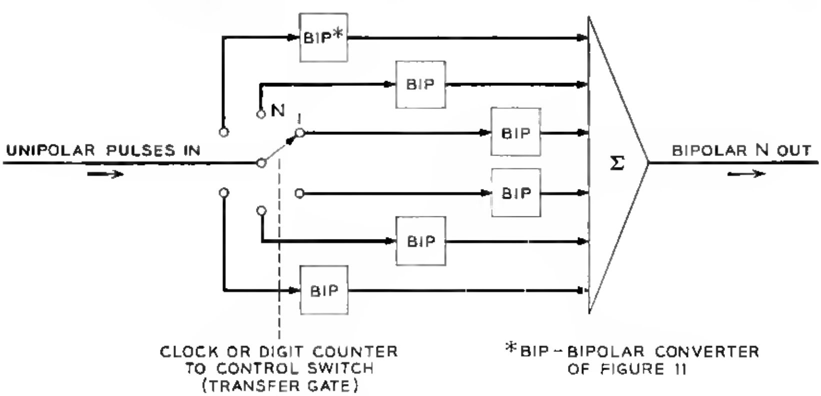

Fig. 13 — Bipolar N block diagram.

$f_r/6$ is $[1 - \cos(4\pi/6)]/[1 - \cos(2\pi/6)] = 3$ times that of bipolar, thereby making compatability between N carrier and PCM more difficult. For 100 per cent duty cycle pulses, complete retiming and width control are even more difficult, and the interference into N carrier is increased by another factor of 4* over 50 per cent duty cycle bipolar. All of the above factors point to bipolar as the preferable scheme provided that the timing jitter accumulation is satisfactory for the system lengths under consideration. This turns out to be the case.[3]

From the form of (15) it can be seen that Meacham's twinned binary can be extended to realize the same spectrum simply by delaying the unipolar train N time slots prior to subtraction. The reconversion at the receiver consists of N parallel converters, each similar to the twinned binary converter. This is an extension of the previous classification; i.e., bipolar-N involves N parallel digital converters at the transmitting terminal and a single analogue restorer at the receiver, while the extension of Meacham's twinned binary consists of an analogue operation at the transmitting terminal and N parallel digital restorers at the receiving terminal.

5.5 *Reduction of Adverse Pulse Density Penalty*

Before we go on to summarize the various pseudo-ternary codes given above, and their realization, we should pause to consider methods for reducing the adverse pulse density penalty. Several means are available, some of which have already been indicated. They include:

1. Use of a slicer to prevent interference from entering the tuned circuit in the absence of a desired pulse.
2. Addition of a timing wave at a spectral null.
3. Use of a very high Q timing circuit to integrate over a long time interval.
4. Addition of noise or a tone in an idle channel.
5. Homogenizing tbe code by using alternate interchange.[15]

The first three techniques given above relate to repeater design, while the last two methods involve terminal processing to minimize the problem at the source. Addition of noise or a tone to an idle channel or alternate interchange are believed to be too expensive for this application. Furthermore, these approaches will not be useful for data transmission. Item 3 above is included to point out that the adverse pulse patterns are transient effects that can be smoothed out by narrow timing filters. Too high a Q is precluded by previous considerations of stability of the tuned

* This assumes that the transmitted pulse peaks are identical in both cases.

circuit. The remaining items have been considered uuder bipolar and interleaved bipolar respectively.

5.6 *Other Timing Approaches*

Use of the dual-mode scheme (listed in Table I), witb timing information transmitted in only one direction, can in principle avoid the crosstalk timing problem. However, delay differences between pairs requires delay build-out of tbe various links, which is akin to the magnitude of the pair selection problem. Similarly, we have devised separate pair timing schemes in which the NEXT problem, as far as timing is concerned, is ideally eliminated, and of course pattern jitter is removed. Again this implies the provision of several clock phases to account for differences in delay among the pairs in the cable. Variations in frequency and phase among the terminals and the transmitted clock will reduce tolerance to pulse crosstalk that obscures the decisiou-making process. In addition, an economic penalty results from utilizing a separate pair for timing. While it is possible to reduce the economic penalty by sharing a single clock among several pairs, this introduces a reliability problem that again is translated into increased cost. For tbese reasons a self-timed approach is more suitable.

5.7 *Choice*

Other self-timing approaches have been considered for this application. Some are variants of the schemes discussed, and all are inferior in most respects to those previously enumerated.

From the standpoint of timing alone, "idealized" interleaved bipolar with timing wave added at $f_r/2$ is the best approach. Wben we consider the problems of realization, ecouomics, compatability with N carrier, and crosstalk into the information-bearing path this approach falls down the preference ladder below bipolar.

Bipolar is chosen for this application since it has tbe following advantages:

1. Energy for timing comes from a region of lower attenuation and higher crosstalk loss than in unipolar.

2. Low-frequency suppression problems are minimized, thereby relaxing transformer requirements and cost.

3. Slicing in the timing path reaps two rewards, namely (a) low-level interference in the absence of a pulse is removed; this reduces tbe adverse pulse density penalty; and (b) pulses exciting the tuned circuit are narrower; consequently finite pulse width effects are reduced.

4. Turnover problems associated with connecting the unbalanced repeater to the balanced line are eliminated.

5. A simple error meter can and has been devised to count bipolar violations and monitor the state of health of the line.

6. Relative compatability with N carrier is as good or better than in other schemes considered.

Most important, however, is the fact that all of these features plus complete retiming and pulse width control can be translated into a physical repeater with presently available components.

VI. PULSE SHAPING (EQUALIZATION)

Most of the foregoing material has dwelt on the problem of timing in self-timed repeaters subject to severe crosstalk interference. Even in the absence of timing interference, it should be clear that crosstalk interference will reduce the ability of a repeater to make a pulse-no pulse decision. Furthermore, the effect of crosstalk in causing errors will be dependent upon the equalization employed in the repeater. In reality, the effects of crosstalk on both timing and pulse recognition cannot be divorced. Ideally, we would like to be able to synthesize the complete repeater structure from input-output specifications. Unfortunately, the present state of the art is such that this is not possible. For this reason we address ourselves to a simpler question. What pulse shaping (both linear and nonlinear) should be employed such that a perfectly timed threshold detector can operate on the incoming signal to reconstruct the desired signal with a minimum probability of error? Since the statistics of the interference are not sufficiently well known, we cannot answer this question. Therefore, we lower our sights further and specialize to the case where the over-all equalized medium (cable plus equalization) is linear and the desired pulse presented to the threshold circuit belongs to a specified family. Several pulse shapes are suitable: i.e., linear phase low-pass Gaussian (LPG), raised cosine, or time response of maximally flat delay transfer function. We will use the LPG simply because the analysis is more tractable and because it can be approached with realizable circuitry. If we also use the lumped capacitor and single interferer to represent the crosstalk interference, we can (subject to certain simplifications) determine that member of the LPG family that minimizes the error probability for a specified coupling. Alternatively, we can consider the "worst-case eye" assumed by Mayo.[3] Under this condition, we can find the parameter of the LPG characteristic such that the sum of intersymbol interference plus peak crosstalk is a minimum for a preselected

crosstalk loss. An LPG characteristic which is 6 db down at half the pulse repetition frequency gives close to the optimum performance with an allowable crosstalk loss of about 30 db that just closes the eye. Details of this exercise in the application of the Fourier transform are not covered herein. The analysis is straightforward but messy. With the addition of low-frequency suppression, mistuning, finite width of the sampling pulse and threshold variations to intersymbol interference, it is not surprising that the allowable equivalent crosstalk loss must be raised to about 35 db to just cause errors in the real life repeater. This allowable coupling must be increased for lines longer than nominal and other factors discussed in Mayo's paper. In passing we note that the actual equalized pulse at the output of the repeater under nominal conditions agrees closely with the response of the optimum LPG characteristic to a 50 per cent duty cycle rectangular pulse.

The spectrum of the bipolar pulse train peaks up at odd integer multiples of $f_r/2$. Peaking in the neighborhood of $(3/2)f_r$ can be troublesome because energy in this region transmitted via the crosstalk path can beat with the desired timing component in the rectifier to produce additional interference at the timing frequency. Therefore, it is desirable to reduce the preamplifier gain in this region. This feature is included in the repeater.[3] Attendant to this modification is a slight change in intersymbol interference. The most pronounced advantage of this feature occurs for dense interfering pulse trains.

In principle, equalization in the timing path can profitably differ from that employed in the information path in the repeater. By the analysis outlined in the Appendix (Section A.2), it can be concluded that the pulses presented to the rectifier should be essentially the same as those giving optimum performance in the information path if the criterion is that the desired bit rate component at the rectifier output be twice that of the undesired component. This result is based on a random crosstalk interferer through a capacitive path and a desired pulse train that is random. In addition, this analysis permits us to conclude that under random conditions, crosstalk into the timing path should not be limiting, but crosstalk effects on the information bearing path are limiting. This improvement over the half bit rate figure used in Section V is due to the different spectral content of the desired pulse train and the crosstalking train that enter the square law rectifier assumed in the analysis. In this regard the bipolar repeater is superior to interleaved bipolar with timing wave added. Furthermore, it should be realized that this advantage is even greater for a dense desired pulse train and a sparse interferer. On the other hand, for adverse pulse

densities in periodic patterns the converse is true — timing becomes limiting before the pulses are obscured by crosstalk. These conclusions agree with experiment.

Before we bring this paper to its conclusion, it might be worthwhile to make a few remarks about our philosophy for choosing a transmission scheme for this application. In many respects we have focused our attention on worst cases. Attendant to this approach is the inherent danger of over-engineering. That we are free from this accusation follows from the fact that the resulting repeater satisfies economic objectives, size requirements, and permits considerable growth of the exchange plant. Furthermore, even if we neglect timing interference, other factors would dictate a repeater of virtually the same complexity. Indeed a carefully designed unipolar repeater is not significantly simpler than the bipolar repeater and suffers more loss in margin due to low-frequency suppression.

There has been a tendency in the literature on the design of PCM repeaters to consider only the mean square value of each source of interference. This is generally followed by the addition of the mean square values of all of the sources of interference and a statement that the resulting distribution of the sum is normal with mean square value given by the sum of the respective mean square values. In effect, the central limit theorem is invoked by incantation, not by proof. Furthermore even if the resulting distribution is normal in the *neighborhood* of the mean, this says nothing about the behavior in the neighborhood of the tails. This is the region of interest in most high-quality pulse systems. In point of fact it can be shown that most of the sources of interference considered herein have distributions that deviate considerably from the normal and are skewed adversely. We will demonstrate this for timing jitter due to mistuning and finite pulse width in another paper.[17] Therefore the treatment of the sum of the sources of interference as being normal involves a dangerous pitfall that we have studiously avoided.

VII. DISCUSSION

With the exception of some model building in the Appendix (Section A.2) we have dealt with essentially deterministic extrapolations rather than with true stochastic models of the real world. As noted previously, this simplification was essential in order to proceed with the development. Subject to these and other assumptions we have shown that the bipolar transmission scheme is well suited to the exchange plant environment. With a random crosstalker and a random desired signal, bipolar has a crosstalk figure of merit for worst case line loss about the same as

TPC, about 35 db. This figure is about 23 db better than that for unipolar under the same conditions. In addition, slicing in the timing path goes a long way toward eliminating the adverse pulse density penalty of 17 db.

The crosstalk figure of merit of 35 db is well below the 1 per cent loss point on the distribution given in Fig. 7. Despite the preliminary nature of these data, this crosstalk figure indicates that it certainly is possible to place a single PCM system within a unit of a unit-constructed cable with no pair selection. Indeed it should be possible to accommodate several systems. Neither of these statements can be made for unipolar. However, the question of how many systems can be installed cannot be answered quantitatively from the work reported in this paper. The multi-system performance is dependent upon how the crosstalking interferers add. This difficult statistical problem is presently being considered both analytically and experimentally by the Systems Engineering Department at Bell Telephone Laboratories. Fortunately, there are a host of techniques that can be employed to further improve the crosstalk situation. They all imply an economic penalty and involve special system layouts. For example, shorter repeater spacing and staggering repeater locations for different directions of transmission can appreciably reduce crosstalk interference. Of course, where it is possible to use separate units or separate cables for the two directions of transmission, this will significantly decrease the probability of crosstalk-induced failure.

We can add nothing to what we have already said about compatability with N carrier. This question is still under study. Here again special measures can be adopted to combat the problem.

With regard to impulse noise, extensive measurements made by T. V. Crater indicate that this should not be limiting provided that the repeater spacing from an office is about half of that of the normal line repeaters.

In short, further consideration of the interaction of the plant with the bipolar repeater is receiving considerable attention by the Systems Engineering Department at Bell Laboratories. Therefore a more quantitative picture will have to await the outcome of their work.

VIII. CONCLUSION

Based largely on timing considerations we have shown that the conventional unipolar PCM transmission scheme is unsuited to the Exchange Plant where crosstalk interference is the principal transmission deterrent. Several pulse transmission schemes have been examined for

replacement of unipolar. We have chosen a bipolar transmission scheme and a self-timed reconstructive repeater incorporating nonlinear timing wave extraction with complete retiming and pulse width control. Reasons for making this choice have been covered in Section V. For random pulse trains on the disturbed and disturbing pairs, bipolar virtually eliminates crosstalk timing interference as a contributor to failure of a repeater.

IX. ACKNOWLEDGMENTS

Many of the concepts and ideas put forth in the paper arose out of useful discussions with J. S. Mayo and E. E. Sumner. It is a pleasure to acknowledge their contributions to this effort.

APPENDIX

Bipolar Through a Square Law Device

A.1 *Random Signal Alone*

The signal transmitted in bipolar can be represented by

$$y(t) = \sum_{n=-\infty}^{\infty} a_n g(t - nT) \tag{16}$$

where a_n is a random variable taking on the values -1, 0, 1 with a priori probabilities $p/2$, $1 - p$, and $p/2$ respectively. It is readily shown that ave $y(t)$ is zero. When this pulse train is put through a square law device, we have at its output

$$\begin{aligned} y^2(t) &= \left[\sum_{n=-\infty}^{\infty} a_n g(t - nT) \right]^2 \\ &= \sum a_n^2 g^2(t - nT) + \sum_n \sum_{\substack{m \\ n \neq m}} a_n a_m g(t - nT) g(t - mT). \end{aligned} \tag{17}$$

The first term above is due to the squares of individual pulses, while the second term arises due to pulse overlaps. The average value of the squared pulse train is

$$\begin{aligned} \text{ave } y^2(t) = \sum R(0) g^2(t - nT) \\ + \sum_n \sum_{\substack{n+k \\ k \neq 0}} R(k) g(t - nT) g(t - (n + k)T) \end{aligned} \tag{18}$$

with

$$R(k) = \text{ave } a_n a_{n+k} \,. \tag{19}$$

That the (average) output of the square law device is periodic may be shown by replacing t by $t + T$ in (18) to get

$$\begin{aligned} \text{ave } y^2(t + T) &= \sum_n R(0) g^2(t - (n - 1)T) \\ &\quad + \sum_n \sum_{\substack{n+k \\ k \neq 0}} R(k) g(t - (n - 1)T) g(t - (n - 1 + k)T) \\ &= \text{ave } y^2(t) \end{aligned} \tag{20}$$

with appropriate changes in summation indices.

Since the ensemble average is periodic, it can be expanded in a Fourier series. Following the same procedure used by Bennett[2] we get

$$\text{ave } y^2(t) = \sum_{n=-\infty}^{\infty} C_m \exp (j2m\pi f_r t) \tag{21}$$

with

$$C_m = f_r \left[R(0) \int_{-\infty}^{\infty} g^2(u) e^{-j2\pi m f_r u} \, du + \sum_k{}' R(k) \int_{-\infty}^{\infty} g(u) g(u + kT) e^{-j2\pi m f_r u} \, du \right] \tag{22}$$

The prime on the summation over k indicates that the $k = 0$ term is to be omitted.

Our main interest resides in the component at the bit frequency, namely C_1 or C_{-1}. By using the relationship between the Fourier trans form of the product of two functions and the convolution of their transforms, we obtain

$$C_1 = f_r \left[R(0) \int_{-\infty}^{\infty} G(-f) G(f_r + f) \, df + \sum_k{}' R(k) \int_{-\infty}^{\infty} G(-f) G(f_r + f) e^{j2\pi k f T} \, df \right]. \tag{23}$$

Since $R(k) = R(-k)$, (23) becomes

$$C_1 = f_r \int_{-\infty}^{\infty} G(-f) G(f_r + f) \left[R(0) + 2 \sum_{k=1}^{\infty} R(k) \cos 2\pi k f T \right] df. \tag{24}$$

Multiplying numerator and denominator of the integral by $G(f)$ gives

$$C_1 = \int_{-\infty}^{\infty} \frac{G(f_r - f)}{G(-f)} P_3(f) \, df \tag{25}$$

where we have taken advantage of the fact that $P_3(f) = P_3(-f)$ and

$$P_3(f) = f_r \mid G(f) \mid^2 \left\{ R(0) + 2 \sum_{k=1}^{\infty} R(k) \cos 2\pi k f T \right\}. \quad (26)$$

It will be recalled from the text that $P_3(f)$ is the label attached to the bipolar spectrum. That the power spectrum of bipolar is given by (26) follows from the general relationship derived for the spectrum of a digital source given by Bennett[7] or from Wolds' theorem for the power spectrum of a stationary time series.[16] Alternatively it can be shown that

$$\begin{aligned} R(k) &= (-1)^{|k|} p^2 (2p - 1)^{|k|-1} \mid k \mid > 1 \\ R(1) &= -p^2 \\ R(0) &= p. \end{aligned} \quad (27)$$

Substitution of these expressions for $R(k)$ in (26) yields the expression for $P_3(f)$ given in (12).

To give a graphic picture of the frequency region that contributes most to the bit frequency component, we make the following assumptions:

1. The pulse shape is Gaussian with transform given by the low-pass Gaussian characteristic below

$$G(f) = \exp\left[-0.693 \left(\frac{f}{f_6} \right)^2 \right] \quad (28)$$

where f_6 is defined as the frequency at which the response is 6 db down from its low-frequency asymptote.

2. $p = 1/2$; therefore $R(k) = 0$ for $\mid k \mid > 1$. Under these assumptions (25) becomes

$$C_1 = \frac{f_r}{2} \int_{-\infty}^{\infty} \exp\left\{ \frac{-0.193}{f_6^2} [(f - f_r)^2 + f^2] \right\} (1 - \cos 2\pi f T) \, df. \quad (29)$$

The integrand of (29) (neglecting a multiplicative constant) is plotted in Fig. 14 for $f_6 = f_r/2$. From this figure it is apparent that the major contribution to the integral comes from the neigbborhood of $f_r/2$.

A.2 *Random Signal Plus Random Crosstalkers*

If we consider the addition of several crosstalkers to the desired signal, the composite signal exciting the square law characteristic is

$$z(t) = y(t) + x(t) \quad (30)$$

where $y(t)$ is given by (16) and the crosstalk interference $x(t)$ is given by

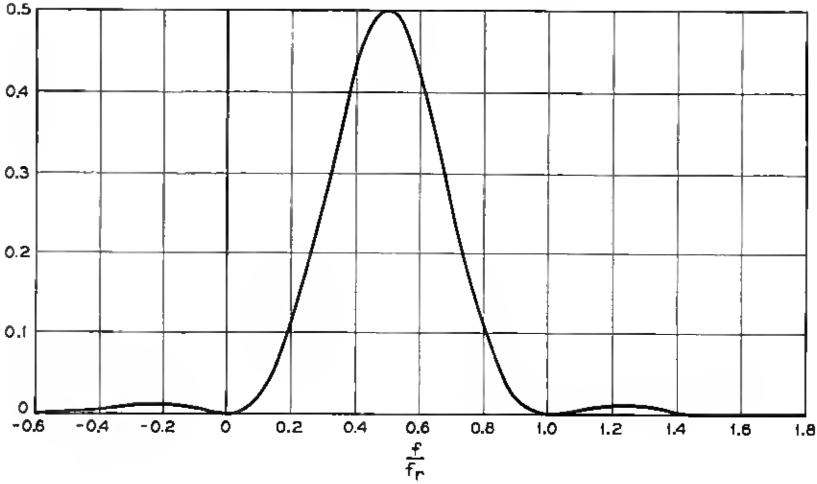

Fig. 14 — Integrand of (25) (except for a constant).

$$x(t) = \sum_{i=1}^{N} \sum_{n=-\infty}^{\infty} b_{in} h_i(t - nT_i - \tau_i) \tag{31}$$

for N interferers. It is assumed that the b_{in} are independent random variables with identical distributions, each distributed in the same manner as the a_n of the desired message. The function $h_i(t)$ is the response of the ith crosstalk path and the preamplifier of the disturbed repeater to a PCM pulse; τ_i is a measure of the phase difference between the ith crosstalk pulse train and the desired message; and T_i is the reciprocal of the pulse repetition frequency of the ith pulse train. If we let $T_i = T + \Delta_i$, then Δ_i is a measure of the difference in pulse repetition frequency between the ith crosstalker and the desired message. In general $h_i(t)$, τ_i, and Δ_i are random variables with the latter two being slowly time varying. We will assume the same frequency for all interferers; therefore $\Delta_i = 0$ for all i. When we average $z(t)$ over the a and b ensembles, we find that its average value is zero as expected. At the output of the square law device, the average value of $z^2(t)$ is

$$\text{ave } z^2(t) = \text{ave } y^2(t) + \text{ave } x^2(t) \tag{32}$$

since $x(t)$ and $y(t)$ are independent random variables with zero mean (with respect to averaging over the message ensembles). By the same argument used previously, ave $z^2(t)$ is periodic with period T and can

be expanded in a Fourier series. The component at the pulse repetition frequency, d_1, is

$$d_1 = C_1 + C_{1x} \tag{33}$$

where

$$C_{1x} = \sum_{i=1}^{N} e^{j2\pi f_r \tau_i} \int_{-\infty}^{\infty} \frac{H_i(f - f_r)}{H_i(-f)} P_{3i}(f)\, df \tag{34}$$

and C_1 is given by (24). $P_{3i}(f)$ is the power spectrum of the ith crosstalker at the input to the square law device, and is given by $P_3(f)$ when $H_i(f)$ is substituted for $G(f)$.

Let us assume that each $H_i = A_i H$. In this context, H is the transform of a "representative" crosstalk pulse and the A_i are random variables. With this relationship substituted in (33), we get

$$\begin{aligned} C_{1x} = \sum_{i=1}^{N} A_i^2 e^{j2\pi f_r \tau} f_r \\ \cdot \int_{-\infty}^{\infty} H(f_r - f) H(f)[R(0) + 2 \sum R(k) \cos 2\pi k f T]\, df. \end{aligned} \tag{35}$$

From (33) we can write the average timing wave component as

$$w(t) = 2 \mid C_1 \mid \cos (2\pi f_r t + \theta_1) + 2 \mid C_{1x} \mid \cos (2\pi f_r t + \theta_x) \tag{36}$$

where θ_1 and θ_x are the angles of C_1 and C_{1x} respectively. The representation of (34) enables us to define an "equivalent crosstalk interferer," namely

$$x(t)_e = \left| \sum_{i=1}^{N} A_i^2 e^{j2\pi f_r \tau_i} \right|^{1/2} \sum b_n h(t - nT - \tau). \tag{37}$$

In (37) τ is a random variable chosen to have the same distribution as the random component of $\theta_x/2\pi f_r$. Of major interest is the magnitude of the equivalent interferer. If we assume that $H(f)$ represents the frequency dependence of the "smooth" crosstalker as modified by the characteristic of the equalizer of the disturbed repeater, then we can assume that the A_i are random variables with the distribution of Fig. 7 that serve to change the level of the smooth characteristic. This model is useful primarily to give some idea as to how the crosstalkers combine to interfere with the desired timing component. We are still left with the difficult problem of determining the distribution of a complicated function of approximately log-normally distributed random variables in order to determine how the ensemble of average timing waves varies

with crosstalk interference. This of course does not indicate how the crosstalk interferers add to confuse the threshold circuit in the repeater.

Again we must lower our sights and revert back to the equivalent capacitive coupling in order to get some feel for how equalization affects the timing path. For an over-all LPG characteristic we can compute the allowable crosstalk coupling such that the ratio of desired to undesired timing component is 2 corresponding to a maximum of 30° phase shift in the tuned circuit; i.e. $| C_1 | / | C_{1x} | = 2$. The maximum allowable crosstalk will be a function of the equalization as characterized by the 6-db point on the LPG characteristic. Evaluation of the integrals for C_1 and C_{1x} (as modified for the deterministic capacitive coupling) shows that the optimum is rather broad and occurs in the neighborhood of $f_6 = f_r/2$. Furthermore the allowable crosstalk coupling is a few db larger than the allowable crosstalk coupling to close the eye in the absence of timing interference. Therefore under random pulse conditions, timing is not limiting.

Similar computations can be made for periodic pulse patterns. This has been done to check the experimental results given by Mayo[3] in his Fig. 57. Analytical results are within 2 db of measured results for the patterns that were spot checked.

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